

Some (typical) machine learning applications

	x (input)	y (output)
Spam detection	document	spam or not
Sentiment analysis	product review	sentiment
Medical diagnosis	patient data	diagnosis
Credit scoring	financial history	loan decision

The cases (input–output) pairs are assumed to be *independent and identically distributed* (i.i.d.).

Statistical Natural Language Processing

Sequence learning

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Structured prediction

In many applications, the i.i.d. assumption is wrong

	x (input)	y (output)
POS tagging	word sequence	POS sequence
Parsing	word sequence	parse tree
OCR	image (array of pixels)	sequences of letters
Gene prediction	genome	genes

Structured/sequence learning is prevalent in NLP.

In this lecture ...

- Hidden Markov models (HMMs)
- A short note on graphical probabilistic models
- Alternatives to HMMs (briefly): HMEM / CRF

... and soon

- Recurrent neural networks

Recap: chain rule

We rewrite the relation between the joint and the conditional probability as

$$P(X, Y) = P(X | Y)P(Y)$$

We can also write the same quantity as,

$$P(X, Y) = P(Y | X)P(X)$$

In general, for any number of random variables, we can write

$$P(X_1, X_2, \dots, X_n) = P(X_1 | X_2, \dots, X_n)P(X_2, \dots, X_n)$$

Recap: (conditional) independence

If two variables X and Y are independent,

$$P(X | Y) = P(X) \quad \text{and} \quad P(X, Y) = P(X)P(Y)$$

If two variables X and Y are independent given another variable Z ,

$$P(X, Y | Z) = P(X | Z)P(Y | Z)$$

An example: probability of a sentence

$P(\text{It's a beautiful day}) = ?$

- We cannot just count all occurrences of the sentence, and divide it to the total number of sentences in English
- But we can base its probability to the probabilities of the words. Using chain rule

$$\begin{aligned} P(\text{It's a beautiful day}) &= P(\text{day} | \text{It's a beautiful})P(\text{It's a beautiful}) \\ &= P(\text{day} | \text{It's a beautiful})P(\text{beautiful} | \text{It's a})P(\text{It's a}) \\ &= P(\text{day} | \text{It's a beautiful})P(\text{beautiful} | \text{It's a})P(a | \text{It's})P(\text{It's}) \end{aligned}$$

- Did we solve the problem?

Markov chains

calculating probabilities

Given a sequence of events (or states), q_1, q_2, \dots, q_t ,

- In a *first-order* Markov chain probability of an event q_t is

$$P(q_t | q_1, \dots, q_{t-1}) = P(q_t | q_{t-1})$$

- Sometimes this equality is just an assumption
- In higher order chains, the dependence of history is extended, e.g., second-order Markov chain:

$$P(q_t | q_1, \dots, q_{t-1}) = P(q_t | q_{t-2}, q_{t-1})$$

Markov chains

definition

A Markov model is defined by,

- A set of states $Q = \{q_1, \dots, q_n\}$
- A special start state q_0
- A transition probability matrix

$$A = \begin{bmatrix} a_{01} & a_{02} & \dots & a_{0n} \\ a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \text{ where } a_{ij} \text{ is the probability of transition from state } i \text{ to state } j$$

Back to sentence probability example

- With a first-order Markov assumption,

$$P(\text{It's a beautiful day}) = P(\text{day} | \text{It's a beautiful})P(\text{beautiful} | \text{It's a})P(a | \text{It's}) \\ = P(\text{day} | \text{beautiful})P(\text{beautiful} | a)P(a | \text{It's})P(\text{It's} | \text{S})$$

- Now the probabilities are easier to calculate
- The above approach is an example of *n-gram language models* that we will return very soon

Hidden/latent variables

- In many machine learning problems we want to account for unobserved/unobservable *latent* or *hidden* variables
- Some examples
 - 'personality' in many psychological data
 - 'topic' of a text
 - 'socio-economic class' of a speaker
- In most structured learning problems, the 'structure' is a hidden variable
- Latent variables make learning difficult: since we cannot observe them, how do we set the parameters?

Learning with hidden variables

(Another) informal/quick introduction to the EM algorithm

- The EM algorithm (or its variants) is used in many machine learning models with latent/hidden variables

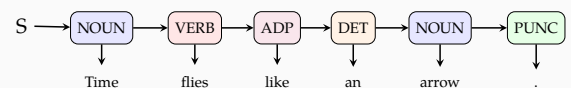
1. Randomly initialize the parameters
2. Iterate until convergence:
 - E-step compute likelihood of the data, given the parameters
 - M-step re-estimate the parameters using the predictions based on the E-step

Hidden Markov models (HMM)

- HMMs are like Markov chains: probability of a state depends only a limited history of previous states
- $$P(q_t | q_1, \dots, q_{t-1}) = P(q_t | q_{t-1})$$
- Unlike Markov chains, state sequence is hidden, they are not the observations
 - At every state q_t , an HMM *emits* an output, o_t , whose probability depends only on the associated hidden state
 - Given a state sequence $q = q_1, \dots, q_T$, and the corresponding observation sequence $o = o_1, \dots, o_T$,

$$P(o, q) = p(q_1) \prod_{t=2}^T P(q_t | q_{t-1}) \prod_{t=1}^T P(o_t | q_t)$$

Example: HMMs for POS tagging



- The tags are hidden
- Probability of a tag depends on the previous tag
- Probability of a word at a given state depends only on the current tag

HMMs: formal definition

An HMM is defined by

- A set of state $Q = \{q_1, \dots, q_n\}$
 - The set of possible observations $V = \{v_1, \dots, v_m\}$
 - A transition probability matrix
- $$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \text{ } a_{ij} \text{ is the probability of transition from state } q_i \text{ to state } q_j$$
- Initial probability distribution $\pi = \{P(q_1), \dots, P(q_n)\}$
 - Probability distributions of
- $$B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{bmatrix} \text{ } b_{ij} \text{ is the probability of emitting output } o_i \text{ at state } q_j$$

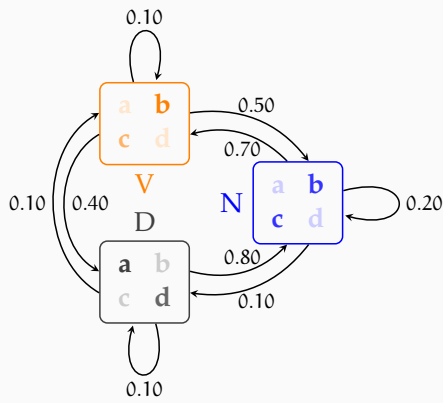
A simple example

- Three states: N, V, D
- Four possible observations: a, b, c, d

$$A = \begin{bmatrix} N & V & D \\ 0.2 & 0.7 & 0.1 \\ 0.5 & 0.1 & 0.4 \\ 0.8 & 0.1 & 0.1 \end{bmatrix} \begin{matrix} N \\ V \\ D \end{matrix} \quad B = \begin{bmatrix} N & V & D \\ 0.1 & 0.1 & 0.5 \\ 0.4 & 0.5 & 0.1 \\ 0.4 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.3 \end{bmatrix} \begin{matrix} a \\ b \\ c \\ d \end{matrix}$$

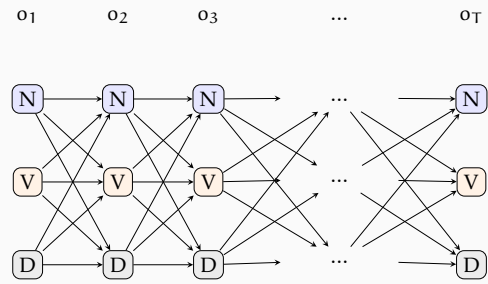
$$\pi = (0.3, 0.1, 0.6)$$

HMM transition diagram



Unfolding the states

HMM lattice (or trellis)



HMMs: three problems

Evaluation

Calculating likelihood of a given sequence

$$P(\mathbf{o} | M)$$

Recognition/decoding

Calculating probability of state sequence, given an observation sequence

$$P(\mathbf{q} | \mathbf{o}; M)$$

Learning

Given observation sequences, a set of states, and (sometimes) corresponding state sequences, estimate the parameters $(\pi, \mathbf{A}, \mathbf{B})$ of the HMM

Assigning probabilities to observation sequences

$$P(\mathbf{o} | M) = \sum_{\mathbf{q}} P(\mathbf{o}, \mathbf{q} | M)$$

- We need to sum over an exponential number of hidden state sequences
- The solution is using a dynamic programming algorithm
 - for each node of the trellis, store *forward probabilities*

$$\alpha_{t,i} = \sum_j \alpha_{t-1,j} P(q_i | q_j) P(o_i | q_i)$$

Assigning probabilities to observation sequences

the forward algorithm

- Start with calculating all forward probabilities for $t = 1$

$$\alpha_{1,i} = \pi_i P(o_1 | q_i) \quad \text{for } 1 \leq i \leq N$$

store the α values

- For $t > 1$,

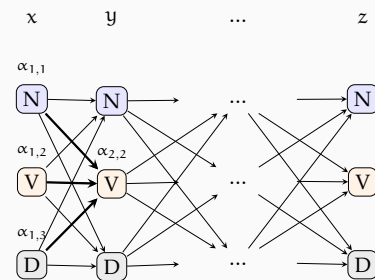
$$\alpha_{t,i} = \sum_{j=1}^N \alpha_{t-1,j} P(q_i | q_j) P(o_i | q_i) \quad \text{for } 1 \leq i \leq N, 2 \leq t \leq T$$

- Likelihood of the observation is the sum of the forward probabilities of the last step

$$P(\mathbf{o} | M) = \sum_{i=1}^N \alpha_{i,T}$$

Forward algorithm

HMM lattice (or trellis)



$$\alpha_{1,1} = \pi_N b_{xN}$$

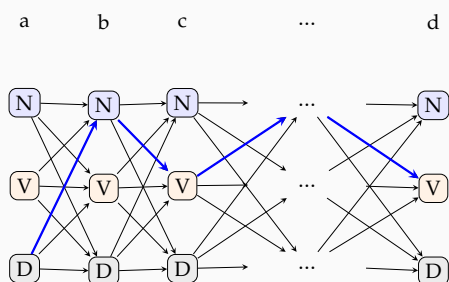
$$\alpha_{2,2} = \alpha_{1,1} a_{NV} b_{yV} + \alpha_{1,2} a_{VV} b_{yV} + \alpha_{1,3} a_{DV} b_{yV}$$

Determining best sequence of latent variables

Decoding

- We often want to know the hidden state sequence given an observation sequence, $P(\mathbf{q} | \mathbf{o}; M)$
 - For example, given a sequence of tokens, find the most likely POS tag sequence
- The problem (also the solution, the *Viterbi algorithm*) is very similar to the forward algorithm
- Two major differences
 - we store maximum likelihood leading to each node on the lattice
 - we also store backlinks, the previous state that leads to the maximum likelihood

HMM decoding problem



Learning the parameters of an HMM

supervised case

- We want to estimate π, A, B
- If we have both the observation sequence \mathbf{o} and the corresponding state sequence, MLE estimate is

$$\pi_i = \frac{C(q_0 \rightarrow q_i)}{\sum_k C(q_0 \rightarrow q_k)}$$

$$a_{ij} = \frac{C(q_i \rightarrow q_j)}{\sum_k C(q_i \rightarrow q_k)}$$

$$b_{ij} = \frac{C(q_i \rightarrow o_j)}{\sum_k C(q_i \rightarrow o_k)}$$

Learning the parameters of an HMM

- Given a training set with observation sequence(s) \mathbf{o} and state sequence \mathbf{q} , we want to find $\theta = (\pi, A, B)$

$$\arg \max_{\theta} P(\mathbf{o} | \mathbf{q}, \theta)$$

- Unlike i.i.d. case, we cannot factorize the likelihood over all observations
- Instead we use EM
 - Initialize θ
 - Repeat until convergence
 - E-step given θ , estimate the hidden state sequence
 - M-step given the estimated hidden states, use 'expected counts' to update θ
- An efficient implementation of EM algorithm is called *Baum-Welch algorithm, or forward-backward algorithm*

HMM variations

- The HMMs we discussed so far are called *ergodic* HMMs: all a_{ij} are non-zero
- For some applications, it is common to use HMMs with additional restrictions
- A well known variant (Bakis HMM) allows only forward transitions



- The emission probabilities can also be continuous, e.g., $p(q|o)$ can be a normal distribution

Directed graphical models: a brief divergence

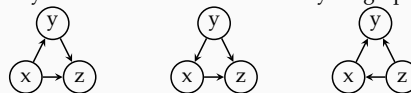
Bayesian networks

- We saw earlier that joint distributions of multiple random variables can be factorized different ways

$$P(x, y, z) = P(x)P(y|x)P(z|x, y) = P(y)P(x|y)P(z|x, y) = P(z)P(x|z)P(y|x, z)$$

- Graphical models* display this relations in graphs,
 - variables are denoted by nodes,
 - the dependence between the variables are indicated by edges

- Bayesian networks are directed acyclic graphs

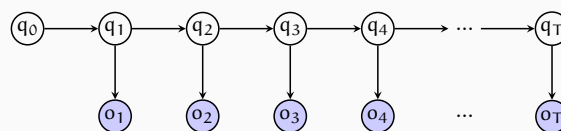


- A variable (node) depends only on its parents

Graphical models

- Graphical models define models involving multiple random variables
- It is generally more intuitive (compared to corresponding mathematical equations) to work with graphical models
- In a graphical model, by convention, the observed variables are shaded
- Graphs can also be undirected, which are called *Markov random fields*

HMM as a graphical model



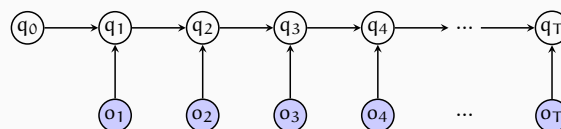
MaxEnt HMMs (MEMM)

- In HMMs, we model $P(\mathbf{q}, \mathbf{o}) = P(\mathbf{q})P(\mathbf{o} | \mathbf{q})$
- In many applications, we are only interested in $P(\mathbf{q} | \mathbf{o})$, which we can calculate using the Bayes theorem
- But we can also model $P(\mathbf{q} | \mathbf{o})$ directly using a *maximum entropy model*

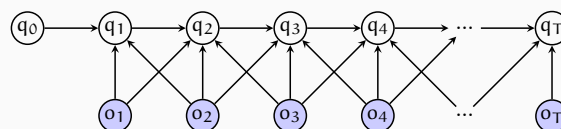
$$P(\mathbf{q}_t | \mathbf{q}_{t-1}, \mathbf{o}_t) = \frac{1}{Z} e^{\sum w_i f_i(\mathbf{o}_t, \mathbf{q}_t)}$$

f_i are features – can be any useful feature
 Z normalizes the probability distribution

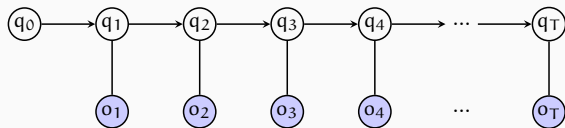
MEMMs as graphical models



We can also have other dependencies as features, for example



Conditional random fields



- A related model used in NLP is *conditional random field* (CRF)
- CRFs are *undirected models*
- CRFs also model $P(\mathbf{q} \mid \mathbf{o})$ directly

$$P(\mathbf{q} \mid \mathbf{o}) = \frac{1}{Z} \prod_t f(q_{t-1}, q_t) g(q_t, o_t)$$

Generative vs. discriminative models

- HMMs are *generative* models, they model the joint distribution
 - you can generate the output using HMMs
- MEMMs and CRFs are *discriminative* models they model the conditional probability directly
- It is easier to add arbitrary features on discriminative models
- In general: HMMs work well when the state sequence, $P(\mathbf{q})$, can be modeled well

Summary

- In many problems, e.g., POS tagging, i.i.d. assumption is wrong
- We need models that are aware of the effects of the sequence (or structure in general) in the data
- HMMs are generative sequence models:
 - Markov assumption between the hidden states (POS tags)
 - Observations (words) are conditioned on the state (tag)
- There are other sequence learning methods
 - Briefly mentioned: MEMM, CRF
 - Coming soon: recurrent neural networks

Next

Mon (after break) sequence learning with neural networks

Have nice break!